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# On the nature of the Hagedorn transition in NCOS systems

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Abstract: We extend the study of the nature of the Hagedorn transition in NCOS systems in various dimensions. The canonical analysis results in a microscopic ionization picture of a bound state system in which the Hagedorn transition is postponed till irrelevancy. A microcanonical analysis leads to a limiting Hagedorn behaviour dominated by highly excited, long open strings. The study of the full phase diagram of the NCOS system using the AdS/CFT correspondence suggests that the microscopic ionization picture is the correct one. We discuss some refinements of the ionization mechanism for $d > 2$ NCOS systems, including the formation of a temperature-dependent barrier for the process. Some possible consequences of this behaviour, including a potential puzzle for $d = 5$, are discussed. Phase diagrams of a regularized form of NCOS systems are introduced and do accomodate a phase of long open strings which disappears in the strict NCOS limit.

Keywords: Duality in Gauge Field Theories, D-branes, AdS-CFT Correspondance, Non-Commutative Geometry.

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## Contents





# 1. Introduction

In this work we study various aspects of the nature of the Hagedorn transition in NCOS systems [1]. There are several bounds in string theory which are of a stringy nature. The first is the string scale $\ell_s$. It was suggested by the $T$-duality symmetry [2] that it is the minimal scale one can probe in string theory; it seems however that with D$p$-branes one may access also smaller distances [3]. The second is the limiting Hagedorn temperature, $T_H$, reflecting an entropy whose leading term, at least for free strings, is linear in the energy in any number of dimensions, corresponding to a gas of highly excited, long strings [4]–[7]. The third one is the critical value of the electric field applied on the brane [8]. It reflects from one point of view the speed of light as a limiting velocity [9]. From another point of view it suggests that the draining of the string tension by the electric field eventually leads to the formation of effectively tensionless strings at the critical value of the electric field [10]. Each of these stop signs attracts an effort to surmount it in hope of uncovering in the process some new fundamental building blocks of matter. This type of wilsonian attitude had been successful in the past. In particular the very same Hagedorn spectrum had provided, in the QCD setting [11], a hint that the hadrons are composite objects, objects which would disintegrate into their constituents once the Hagedorn temperature was approached. The analogous scenario for fundamental strings was set up in ref. [12], although the precise nature of the 'constituent phase' lying beyond the Hagedorn temperature remained mysterious (see [13] for new results in this direction), mostly due to the specific difficulties brought by the presence of strong gravitational effects.

The string/black-hole correspondence principle [14, 15] can be used to address the question of gravitational effects at a qualitative level [16]. In this picture the Hagedorn temperature is effectively the maximal temperature of the system: a Hagedorn phase of long strings appears as a transient that matches at high energies and/or coupling to black holes or black branes of negative specific heat, the Hagedorn temperature being maximal because such black holes are colder the higher the energy. Therefore, the question of the 'string constituents' appearing at very high energies would depend on the appropriate holographic description of microscopic degrees of freedom on the horizon of very large black holes. Such an scenario is actually realized within the AdS/CFT correspondence [17, 18].

One has to keep in mind that the presence of real bounds may actually be a signal that the wilsonian quest for an ultraviolet fixed point should be abandoned in this case. However in the absence of an alternative we will continue the quest for what lies beyond and/or instead of the long-string phase.

The NCOS systems were originally defined as certain limits of D-brane backgrounds in the presence of a near-critical time-space noncommutative parameter. That resulted in open strings that are essentially decoupled from gravity and whose tension is much smaller than that of usual strings, two properties that make NCOS systems ideal testing grounds for the above circle of ideas.

The thermodynamical properties of these systems have been discussed by various authors [19]–[22]. Following these works we concentrate on the question of surpassing the Hagedorn temperature: does one have a phase of long, highly excited strings as the dominant physical states at energies high above the string scale and sufficiently weak coupling. Actually, a conservative approach could be to note that the NCOS system does have a microscopic description in terms of a bound state of D$p$-branes and F1-strings. From such a viewpoint one could expect a rather similar behaviour to that of QCD, i.e. as the temperature approaches the critical one, the system would begin to dissociate into its constituents exposing its bound-state nature. Such a 'deconfinement' was indeed found in [21]. We will study this dissociation in more detail here, by subjecting it to both a canonical and a microcanonical analysis. The results will turn out to be different and we try to reconcile them by drawing the phase diagrams of these systems. We will find that, once again, by and large the system evades a 'real' Hagedorn phase, that is the excitation of long open strings is not attained. In the process we learn about some new features of the ionization mechanism in NCOS systems.

In section 1 we will review in some detail the limiting procedure originally used to define NCOS. One of its features is supposed to be that by appropriately taking



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a strong bulk string coupling limit one ends up with a weakly coupled NCOS. One should bear in mind that open strings are not BPS states and thus the expectation for a weakly coupled Hagedorn spectrum may be unwarranted for. Some useful formulae are collected in this section.

In section 2 we study the thermodynamics of NCOS systems for dimension less than six. The canonical and microcanonical analysis will lead to contradicting physical pictures. According to the canonical analysis the long strings are never excited; as the temperature is increased an ionization process becomes possible. For a temperature of the order of the noncommutative energy scale the system can be reliably studied in weak coupling and it starts emitting the F1 constituents. This is essentially the Hagedorn temperature of the bound state. The emission of a liberated rigid string turns out to have several consequences: it raises the Hagedorn temperature of the remaining bound state, it gently increases the NCOS coupling and it decreases the effective volume of the remaining bound state, thus forming a barrier to ionization for dimension larger than two. All in all, the ionization process dominates and many F1s get liberated. We also touch upon a possibility that the ionization can be reversed into a 'recombination'. This turns out to be potentially possible only in five dimensions, the case whose dual is called OM theory [23]. This behaviour will eventually lead to a puzzle that will be discussed in section 2.2.

According to a microcanonical analysis something totally different will happen. For any energy much larger than the string-scale energy, the typical states consist of long open strings attaching to a non-ionized bound state. This result is obtained once one allows long open strings to contend in the entropy competition.

In section 3 the problem of conflicting behaviours is subjected to a supergravity phase-diagram analysis, the result of which is that the microscopic bound state picture is the correct one. Namely, the matching to supergravity phases via the correspondence principle excludes a standard Hagedorn phase with linear scaling of the entropy. The long open strings are only an effective description which breaks down at a high enough energy. The Hagedorn transition is postponed time and again until the supergravity picture becomes the effective one and the question of the Hagedorn transition becomes mute.

### 1.1 Notation and conventions

In this section, which may be skipped in a first reading, we review the basic properties of NCOS systems and fix our notation.

The $(p+1)$-dimensional NCOS theory [1] is perturbatively defined in terms of the open-string dynamics on a D$p$-brane with an electric field background $\mathcal{E}$, in the critical limit $2\pi\alpha'\,\mathcal{E} \to 1$ [8]. This limit is characterized by the emergence of nearly tensionless open-string excitations. A convenient way of isolating these light strings involves a zero-slope limit $\alpha' \to 0$ in a model with anisotropic sigma-model metric. We parametrize the anisotropy of the background metric in terms of anisotropic

Regge-slope parameters, i.e. we write a world-sheet action:

$$S_{\rm NCOS} = -\frac{1}{4\pi}\int_\Sigma \left(\frac{\eta_{\mu\nu}}{\alpha'}\,\partial X^\mu \partial X^\nu + \frac{\delta_{ij}}{\alpha'_\perp}\,\partial X^i \partial X^j + \frac{\delta_{ab}}{\alpha'_\perp}\,\partial X^a \partial X^b\right) + \mathcal{E}\oint_{\partial\Sigma} X^0 \partial X^1\,, \tag{1.1}$$

where $\mu,\nu = 0,1$, $i,j = 2,\dots,p$ and $a,b = p+1,\dots,9$. The electric field of modulus $\mathcal{E}$ points along the $X^1$ direction.

Following [24], the open-string dynamics is characterized by an effective open-string metric

$$(G_{\rm open})_{\mu\nu} = \frac{\alpha'_\perp}{\alpha'_e}\,\eta_{\mu\nu}\,, \qquad (G_{\rm open})_{ij} = \delta_{ij}\,, \tag{1.2}$$

together with an 'electric' noncommutativity parameter, $[X^0, X^1] = i\,\theta_e$, and an effective coupling $G_o$, given by

$$\theta_e = 2\pi\sqrt{(\alpha'_e)^2 - (\alpha')^2}, \qquad G_o^2 = g_s\sqrt{\frac{\alpha'}{\alpha'_e}}\,, \tag{1.3}$$

where $g_s$ is the nominal string coupling in the bulk and $\alpha'_e$ is the effective Regge-slope parameter in the electric plane:

$$\alpha'_e = \frac{\alpha'}{1-(2\pi\alpha'\mathcal{E})^2}\,. \tag{1.4}$$

In many instances, it is useful to choose coordinates so that $\alpha'_e = \alpha'_\perp$, which makes the effective open-string metric minkowskian. On the other hand, we will also discuss situations where $\alpha'_e/\alpha'_\perp$ is not constant, so that we keep the most general notation in the following, and distinguish between both Regge-slope parameters.

Denoting by $\epsilon = \alpha'/\alpha'_e$ the ratio of effective and sigma-model tensions, the NCOS limit is defined by $\epsilon \to 0$ at fixed effective Regge slope $\alpha'_e$ and fixed effective coupling $G_o$. Notice that this limit involves strong coupling in the asymptotic closed-string background, since $g_s \sim 1/\sqrt{\epsilon} \to \infty$. However, the open strings on the brane should remain weakly coupled provided $G_o \ll 1$ and the world-volume dynamics essentially decouples from the closed strings in the bulk, via a kinematical mechanism. This is the main dynamical property of NCOS theories. It can be argued in perturbation theory, modulo some assumptions, for $p \le 6$ cf. [1] (see, however [25]). At this point, it is worth mentioning that the open strings on the D-brane background serving as a definition of the NCOS system are not BPS states themselves. Since a (bulk) strong coupling limit $g_s \to \infty$ is implied, the reliability of the description in terms of NCOS open strings is not completely guaranteed in all circumstances.

Keeping in mind all these warnings, we have a pure theory of open strings with effective coupling $G_o$ and free spectrum given by the solution of

$$(G_{\rm open})^{\alpha\beta} p_\alpha p_\beta + M^2 = 0\,, \tag{1.5}$$

where $M^2$ is the open-string spectrum in the absence of an electric field. This results in a dispersion relation

$$\omega_p = \sqrt{p_1^2 + \frac{\alpha'_\perp}{\alpha'_e}\,\mathbf{p}_\perp^2 + \frac{N_{\rm osc}}{\alpha'_e}}\,, \tag{1.6}$$

where $\mathbf{p}_\perp$ denotes the momentum in the world-volume directions $X^i$, transverse to the electric field, and $N_{\rm osc}$ is the string oscillator number, including possible world-sheet zero-point energies.

With $N$ parallel D$p$-branes, the effective expansion parameter of NCOS perturbation theory is the stringy 't Hooft coupling:

$$\lambda_o = 2\pi\, G_o^2\, N\,. \tag{1.7}$$

The low-energy spectrum is that of $\mathcal{N}=4$ super–Yang-Mills theory with gauge group U$(N)$ and gauge coupling

$$g_e^2 = (2\pi)^{p-2}\, G_o^2\, (\alpha'_e)^{\frac{p-3}{2}}\,. \tag{1.8}$$

Notice that the effective expansion parameter of the low-energy perturbation theory is the dimensionless combination $g_e^2 N\, E^{p-3}$, with $E$ the typical energy scale. This effective coupling matches $\lambda_o$ at the string scale of the NCOS: $E \sim 1/\sqrt{\alpha'_e}$.

On the other hand, according to (1.6), the high-energy asymptotics of the density of states is controlled by the 'electric' Regge slope $\alpha'_e$:

$$\rho(\omega) \sim \exp\left(\frac{\omega}{T_{He}}\right)\,, \tag{1.9}$$

with $T_{He}$ the effective Hagedorn temperature. For type II strings one has

$$T_{He} = \frac{1}{\sqrt{8\pi^2\alpha'_e}}\,. \tag{1.10}$$

This temperature also sets the scale of noncommutative effects in the NCOS theory.

There is an equivalent 'constituent picture' for this system that is conceptually useful. Namely, we can obtain the constant electric field $\mathcal{E}$ as a condensate of fundamental strings stretched along the $X^1$ direction [26]. Therefore, we have a bound state of $N$ D$p$-branes and $n$ F1-strings. The density of F1-strings is determined in terms of the electric field by the relation

$$\frac{n}{V_\perp} = \frac{\partial \mathcal{L}(\mathcal{E})_{\rm DBI}}{\partial \mathcal{E}}\,, \tag{1.11}$$

where $\mathcal{L}_{\rm DBI}$ is the Dirac-Born-Infeld lagrangian that controls the classical dynamics of constant electric fields:

$$\mathcal{L}(\mathcal{E})_{\rm DBI} = -N\,\mathcal{T}_{{\rm D}p}\,\sqrt{1-(2\pi\alpha'\mathcal{E})^2}\,, \tag{1.12}$$

with the D$p$-brane tension given by:

$$\mathcal{T}_{\mathrm{D}p} = \frac{2\pi}{g_s\,(2\pi\sqrt{\alpha'}\,)^2\,\left(2\pi\sqrt{\alpha'_\perp}\,\right)^{p-1}}\,. \tag{1.13}$$

We find

$$n = \frac{NV_\perp}{\left(2\pi\sqrt{\alpha'_\perp}\right)^{p-1} g_s}\,\frac{2\pi\alpha'\mathcal{E}}{\sqrt{1-(2\pi\alpha'\mathcal{E})^2}} = \frac{NV_\perp}{\left(2\pi\sqrt{\alpha'_\perp}\right)^{p-1} g_s\sqrt{\alpha'}}\,\sqrt{\alpha'_e}\,\sqrt{1-\epsilon}\,. \tag{1.14}$$

These formulae are exact, with the NCOS limit given by $\epsilon \to 0$. An expression for $n$ in terms of *just* the effective parameters of the NCOS is:

$$n = \frac{NV_\perp}{\left(2\pi\sqrt{\alpha'_\perp}\right)^{p-1}}\,\frac{1}{G_o^2}\,\frac{\theta_e}{2\pi\alpha'_e} = \frac{NV_\perp}{\left(2\pi\sqrt{\alpha'_\perp}\right)^{p-1}}\,\frac{1}{G_o^2}\,\sqrt{1-\epsilon}\,. \tag{1.15}$$

In all these relations, $V_\perp$ denotes the *coordinate volume* in the $X^i$ directions. Namely, if we identify periodically $X^i \equiv X^i + L_\perp$, then $V_\perp = (L_\perp)^{p-1}$. Notice that, in general, $V_\perp$ *does not* represent a proper volume in the effective open-string metric, unless $\alpha'_e = \alpha'_\perp$.

Equation (1.14) implies an approximate scaling

$$\alpha'_e \propto n^2 \tag{1.16}$$

in the NCOS regime $\epsilon \ll 1$, at fixed values of the sigma-model parameters. Analogously, (1.15) gives $G_o^2 \propto 1/n$ under the same conditions.

In the bound-state picture, the NCOS regime is characterized by the mass-dominance of the F1-string component, i.e. the exact BPS formula for the mass of the $(N,n)$ bound state is:

$$M_{(N,n)} = \sqrt{M_N^2 + M_n^2} = M_n + \frac{M_N^2}{2M_n} + \mathcal{O}(\epsilon)\,, \tag{1.17}$$

where $M_N = N\,L\,V_\perp\,\mathcal{T}_{\mathrm{D}p}$ is the mass in D$p$-branes and $M_n = n\,L\,\mathcal{T}_{\mathrm{F1}} = nL/2\pi\alpha'$ is the mass in F1-strings. Here, $L$ denotes the *coordinate length* in the $X^1$ direction. Thus, in the NCOS limit

$$M_{(N,n)} = \frac{nL}{2\pi\alpha'} + \frac{1}{4\pi n}\,\frac{N^2 L V_\perp^2}{(4\pi^2\alpha'_\perp)^{p-1} g_s^2\alpha'} + \mathcal{O}(\epsilon). \tag{1.18}$$

This gives a simple formula for the binding energy of a single F1-string in the NCOS limit with large $n$:

$$E_{\mathrm{binding}} = \lim\nolimits_{\mathrm{NCOS}}\left[M_{(N,n-1)} + M_{(0,1)} - M_{(N,n)}\right] = \frac{L}{4\pi\alpha'_e}\,(1+\mathcal{O}(1/n))\,. \tag{1.19}$$

Notice that the resulting binding energy is finite, in spite of the infinite stiffness of the free F1-strings in the NCOS limit. It is interesting to look at the mass hierarchy of other BPS states in this limit. Since $\alpha' \to 0$, the tension of any brane will diverge in the limit, unless it is compensated by an appropriate power of $g_s$. In particular, D$q$-branes stretched in directions orthogonal to the electric field have a mass

$$M_{\mathrm{D}q} = \frac{2\pi\, V_q}{g_s\,(2\pi\sqrt{\alpha'})\,\left(2\pi\sqrt{\alpha'_\perp}\right)^q} \longrightarrow \frac{2\pi\, N\, V_q}{\lambda_o\,\left(2\pi\sqrt{\alpha'_\perp}\right)^q\,\sqrt{\alpha'_e}}\,, \tag{1.20}$$

whereas a NS5-brane stretched along the electric field also survives with a mass

$$M_{\mathrm{NS5}} = \frac{2\pi\, V_5}{g_s^2(2\pi\sqrt{\alpha'})^2\,\left(2\pi\sqrt{\alpha'_\perp}\right)^4} \longrightarrow \frac{2\pi\, N^2\, V_5}{\lambda_o^2\,\alpha'_e\,\left(2\pi\sqrt{\alpha'_\perp}\right)^4}\,. \tag{1.21}$$

## 2. Thermodynamics of NCOS theories

In this section we consider the thermodynamics of NCOS theories at weak NCOS coupling $G_o \ll 1$. We start by reviewing some established facts about thermal ensembles of open strings in various dimensions. Then we review the 'ionization mechanism' of ref. [21] that realizes the Hagedorn phase transition of this system in the canonical ensemble. We also discuss various refinements of the ionization mechanism that are of some importance in the matching to strong-coupling descriptions based on supergravity.

Next, we turn to the microcanonical ensemble and show that it *is not* equivalent to the canonical ensemble. Namely, the ionization process *does not occur* in the microcanonical ensemble. The high-energy regime of the theory, as inferred from the free spectrum, is very similar to more standard open-string systems. At the end of this section we will be left with an apparent contradiction.

### 2.1 Generalities of open-string thermodynamics

For a system of $N$ parallel D$p$-branes, the effective expansion parameter in perturbation theory is the stringy 't Hooft coupling $\lambda_s = g_s\, N$, where $g_s$ is the closed-string coupling. Thus, for large $N$ with fixed and small $\lambda_s$ we have weakly coupled open strings with an ever weaker coupling to closed strings (in NCOS systems the claimed decoupling between closed and open-string sectors does not require large $N$).

At energy densities much larger than the stringy energy density, controlled by the string length scale $\ell_s$, we expect the thermal ensemble to be dominated by highly excited or *long* individual strings with density of states (1.9). The leading term of the microcanonical entropy of such a system is

$$S(E)_{\mathrm{Hagedorn}} \approx \frac{E}{T_H}\,, \tag{2.1}$$

with $T_H \sim 1/\ell_s$. This defines a thermal ensemble with constant temperature $T_H$ and infinite specific heat. The precise character of the thermodynamics depends on the subleading corrections, that are sensitive to dimensionality and finite-size effects [6, 27, 18, 16, 28]. Assuming that the space transverse to the D$p$-brane is infinite (so that it does not support open-string 'winding modes') the critical parameter is the dimensionality of the D-brane. For $p \geq 5$ the energy is shared by a large number long strings and the microcanonical ensemble gives the same result as the canonical ensemble, i.e. $T_H$ is a physical limiting temperature in the sense that it takes infinite energy to reach it. The corrections to (2.1) make the specific heat positive. At very large volumes $VT_H^p \gg 1$ one finds, in terms of the critical Hagedorn energy $E_H \sim N^2\, V\, T_H^{p+1}$:

$$S_{\mathrm{D}p} \approx \frac{E}{T_H} + C_p\, N^2\, \frac{5-p}{7-p}\, V\, T_H^p \left(\frac{E}{E_H}\right)^{\frac{7-p}{5-p}}, \tag{2.2}$$

with two exceptions, the cases of D5- and D7-branes:

$$S_{\mathrm{D5}} \approx \frac{E}{T_H} - C_5\, N^2\, V\, T_H^5\, e^{-E/E_H}, \qquad S_{\mathrm{D7}} \approx \frac{E}{T_H} + C_7\, N^2\, V\, T_H^7 \log\left(\frac{E}{E_H}\right). \tag{2.3}$$

In all cases, the specific heat is positive and the system is extensive.

On the other hand, for $p < 5$ most of the energy flows into a *single* long open string [6] and the resulting entropy law is of the form

$$S_{\mathrm{D}p} \approx \frac{E}{T_H} - C_p \log\left(\frac{E}{E_H}\right), \tag{2.4}$$

with non-extensive leading corrections turning the system into a negative specific heat one. The Hagedorn temperature is non-limiting in the sense that one can reach it with a finite energy density of $\mathcal{O}(N^2)$ in string units. Since the resulting long-string system is thermodynamically unstable in infinite volume, this raises the possibility of a phase transition into a different phase that would exist a higher temperatures. Still, the entropy law (2.4) is perfectly acceptable as the logarithm of the density of states for a finite-volume system. Thus, working in the microcanonical ensemble, at finite total energy $E$, one may try to incorporate directly the interaction effects into the long-string picture.

A consistent picture of the effects of interactions emerges using the string/black-hole correspondence principle as generalized in [15]. The basic assumption here is that highly excited, long open strings on the D$p$-brane world-volume match the properties of black-branes at sufficiently high energy or coupling. Although the correspondence principle strictly applies to single-string states, it is expected to provide a qualitative description of the multi-string gas within $\mathcal{O}(1)$ accuracy in the coefficients, provided we work in the microcanonical ensemble at finite total energy [16].

Highly non-extremal (or Schwarzschild) black $p$-branes have an entropy

$$S_{\text{black}} \sim \frac{E}{T_H} \left(\frac{\lambda_s^2\, E}{N^2\, E_H}\right)^{\frac{1}{7-p}}, \tag{2.5}$$

which matches the Hagedorn entropy at energies of order

$$E_{\rm match} \sim \frac{N^2}{\lambda_s^2}\, E_H\,. \tag{2.6}$$

This point is intrinsically singled out because precisely at these energies the curvature at the horizon of the black brane is $\mathcal{O}(1)$ in string units, i.e. stringy corrections to the semiclassical background metric become of $\mathcal{O}(1)$ at this point. From the point of view of the weak-coupling string perturbation theory, the correspondence point is associated to the collapse of the long string due to self-gravity [29]. Thus, long-string phases with a Hagedorn-type density of states are expected to match at strong coupling to black-brane metrics of Schwarzschild type, i.e. with negative specific heat. In this microcanonical picture, the Hagedorn temperature is approximately maximal for *all values* of $p$, since it is approximately constant (to $\mathcal{O}(1)$ accuracy) within the long-string regime and it is decreasing with the energy in the black-brane regime. Notice that the matching (2.6) is trivialized in the strict large-$N$ limit with fixed $\lambda_s$. If we insist in decoupling the closed-string sector completely, both the standard Hagedorn regime of long strings — starting at energies of $\mathcal{O}(N^2)$, and the black-brane regime run away to infinity. Closed-string decoupling in NCOS theories does not require large $N$, but requires large volume instead.

This picture is markedly different from the one outlined in [21] for the case of NCOS strings. The authors of [21] carry out a canonical analysis with the temperature (rather than the total energy) as control parameter. It is found that NCOS systems can surpass their Hagedorn temperature, $T_{He}$, by dissociating into the 'constituents', i.e. there is ionization of the (D$p$, F1) bound state by F1-string emission. In this process, the effective Hagedorn temperature self-tunes to the temperature of the heat reservoir, so that the total energy density increases according to the rules of the canonical ensemble, without ever exciting a significant number of long open strings in the NCOS bound state. The choice between these two pictures is one of the main themes of this work.

### 2.2 Canonical approach: thermal ionization of F1-strings

Let us now consider the bound system of $N$ D$p$ branes and $n$ F1-strings in the region where the perturbative NCOS description is appropriate, i.e. one has constructed a theory of open strings with an effective string length, $\sqrt{\alpha'_e}$, which is essentially decoupled from closed strings and whose coupling, $G_o \sim 1/\sqrt{n}$, can be made small for a large enough value of $n$. One may expect, based on the discussion of the previous subsection that, precisely for $p < 5$ we can access the effective Hagedorn temperature $T_{He}$ and even surpass it, probing the phase transition, i.e. for $p < 5$ the internal energy, as estimated from the free string approximation, is finite at the Hagedorn temperature [18, 16].

However these light strings are in a sense not elementary; they were constructed by forming a bound state. When the system is heated up it has the option to dissociate and 'melt' into its constituents. An estimate of the feasibility of this melting is obtained by calculating the free energy of a single dissociated F1-string at large $n$. For $LT \gg 1$ we have:

$$F_{\rm F1} = \frac{L}{2\pi\alpha'} - 2\pi\, L\, T^2\,. \tag{2.7}$$

The first term gives the static mass of the F1-string $M_{(0,1)}$, and diverges in the NCOS limit. This justifies considering the ejected F1-string as 'rigid', so that the free energy from thermal fluctuations — the second term, comes from the massless 'Goldstone multiplet', a vector multiplet in $1+1$ dimensions.[1] If we normalize the static energy by the rest mass of the bound state we find

$$\Delta F_{\rm ion} \approx E_{\rm binding} - 2\pi\, L\, T^2 \approx \frac{L}{4\pi\alpha'_e} - 2\pi\, L\, T^2\,. \tag{2.8}$$

The free energy of the bound state will be considered unchanged in this first estimate. Thus, it is the vanishing of the single-string free energy which determines the critical temperature above which the system may 'ionize':

$$T_{\rm critical} = \frac{1}{\sqrt{8\pi^2\alpha'_e}} = T_{He}\,, \tag{2.9}$$

precisely the effective Hagedorn temperature of the NCOS (1.10). Notice that, strictly speaking, the ionization of F1-strings is only possible for finite $L$. In this situation there is no complete decoupling from the closed-string sector [25], the emission of wound F1-strings described here being a good example. On the other hand, we essentially postpone the study of finite-size effects to a future publication [30]. Throughout this paper, we keep only the leading, extensive form of all thermodynamic expressions in the large-volume limit.

The critical ionization temperature was fixed by field-theoretic dynamics. As pointed out in [21], the ionization has the effect of increasing slightly the Hagedorn temperature of the strings attached to the remaining bound state. This occurs because the relation between $\alpha'_e$ and $n$ at fixed values of the bulk moduli is $\alpha'_e \sim n^2$. Thus as $n$ decreases so does $\alpha'_e$, which in turn leads to an increase of $T_{He}$, i.e. the more fundamental strings dissociate, the more the effective Hagedorn temperature rises. After ejecting one F1, $\alpha'_e$ decreases and the effective Hagedorn temperature $T_{He}$ of the remaining $(N, n-1)$ bound state rises accordingly. As the temperature $T$ reaches the new threshold a second F1 is ejected and so on. If we take large $n$

[1] Rigid F1-strings that have been ejected from the bound state are called 'long strings' in [21]. We use 'rigid' in order to avoid confusion with the long (highly excited) open strings on the bound state.

we can view the process as a continuous discharge of F1-strings, in such a way that the system at a given point is a $(N, n')$ bound state, plus $n - n'$ F1-strings, and one can regard the bound state as always sitting at its effective Hagedorn temperature, $T_{He}(n') > T_{He}(n)$. Thus the ionization process postpones the transition of the 'real' Hagedorn temperature, which continuously self-tunes to the temperature of the canonical ensemble. Like a mirage oasis in the desert, the Hagedorn transition continuously receeds as it is approached. The dominant configurations are not those of the long open strings and, although the energy is above the string scale, these configurations do not get activated.

Let us set $\alpha'_e = \alpha'_\perp$ before the leakage begins, so that the open-string metric is minkowskian for the $(N, n)$ bound state, and denote $T_H$ its effective Hagedorn temperature:

$$T_H \equiv T_{He}(n) = \frac{1}{\sqrt{8\pi^2 \alpha'_\perp}}\,. \tag{2.10}$$

Then at any other point we have

$$\frac{\alpha'_e}{\alpha'_\perp} = \left(\frac{n'}{n}\right)^2 = \left(\frac{T_H}{T}\right)^2 . \tag{2.11}$$

In terms of the ionization fraction $x \equiv n'/n$ and normalized temperature $t = T/T_H$, we find

$$x(t) = \frac{1}{t}\,, \qquad \text{for } t > 1\,, \tag{2.12}$$

whereas $x(t) = 1$ for $t < 1$, i.e. before ionization starts.

It is also important to notice that the NCOS coupling of the remaining bound state also becomes temperature-dependent, since $G_o^2 \propto 1/n$. We find for the 't Hooft coupling:

$$\lambda_o(t) = \lambda_o\, t\,, \tag{2.13}$$

with $\lambda_o$ the 't Hooft coupling of the initial $(N, n)$ system. Therefore, the F1-emission process rises the coupling of the 'ionized' NCOS system linearly with the temperature. At temperatures of order

$$T_{\text{strong}} \sim \frac{T_H}{\lambda_o}\,, \tag{2.14}$$

or ionization fraction $x_{\text{strong}} \sim \lambda_o^2$, the D$p$-brane system should become strongly coupled. It will turn out that the Horowitz-Polchinski (HP) correspondence line to supergravity is

$$\lambda_o \sim \frac{1}{t^2} \tag{2.15}$$

universally for $t \gg 1$. Therefore, in the weak-coupling regime $\lambda_o(t) < 1/t \ll 1$ for any temperature large enough, and one must always change variables to supergravity before hitting the limit (2.14).

**A thermal barrier to ionization for $d > 2$.** In fact, it is not only the Hagedorn temperature that is shifted during F1-ionization. As follows from (1.6), each of the momentum modes $\mathbf{p}_\perp$ in the commutative directions get their effective metric rescaled by $\alpha'_e/\alpha'_\perp$. In describing the thermodynamics as a function of termperature, it is convenient to maintain the definition of 'temperature' unchanged. In our case, we measure energies with respect to the time-like Killing vector $\partial/\partial X^0$. Notice that $X^0$ measures proper time in the open-string metric of the $(N, n)$ bound state, but this is no longer true after some F1-strings have been ejected. Thus, in writing the thermodynamic functions of the general $(N, n')$ NCOS theory, we have to take into account the effective rescaling of the metric in (1.6), i.e. they are given by those of a normal string theory with Regge slope $\alpha'_e$ and living in a box of *smaller* effective volume

$$V_{\text{eff}} = L\, V_\perp \left(\frac{\alpha'_e}{\alpha'_\perp}\right)^{\frac{p-1}{2}} = L\, V_\perp \left(\frac{n'}{n}\right)^{p-1} . \tag{2.16}$$

The main consequence of this effective renormalization of the volume is that it significantly affects the free energy of the bound state in the ionization process. This in turn results in the generation of an effective thermal barrier to the activation of the ionization process.

The entropy density of the bound state in the vicinity of the Hagedorn temperature is of $\mathcal{O}(N^2)$ in string units. This is the entropy that comes out of matching the massless-dominated and long-string dominated entropy formulas at the Hagedorn temperature:

$$S_{\text{massless}} = N^2\, C_p\, V\, T^p\,, \tag{2.17}$$

with $C_p$ a function of $\lambda_o$ that is approximately constant at weak coupling:

$$C_p = \frac{8(p+1)\,\text{Vol}(S^{p-1})}{(2\pi)^p} \left(2 - \frac{1}{2^p}\right) \Gamma(p)\, \zeta(p+1) + \mathcal{O}(\lambda_o)\,, \tag{2.18}$$

and

$$S_{\text{long}} = c_s \frac{E}{T_{He}} \tag{2.19}$$

with $c_s = 1 + \mathcal{O}(\lambda_o)$. Thus, we shall write

$$F_{\text{bs}}(x) = -N^2\, C\, V_{\text{eff}}\, T^{p+1} + M_{(N,n')} = -N^2\, C\, V_\perp\, L\, T^{p+1}\, x^{p-1} + M_{(N,n')} \tag{2.20}$$

for the free energy of the bound state in the vicinity of $T \approx T_{He}$. We account for the uncertainty of matching effects by the freedom of choosing the constant $C$ up to an $\mathcal{O}(1)$ factor, to leading order in the weak-coupling expansion.

Adding the free energy of the $n - n'$ ejected F1-strings:

$$F_{\text{F1}} = -2\pi\, L\, T^2\, (n - n') + M_{(0,n-n')} = -2\pi\, L\, T^2\, n\, (1 - x) + M_{(0,n-n')}\,, \tag{2.21}$$

and normalizing by the mass of the initial bound state $M_{(N,n)}$, we find for the function

$$f(x,t) \equiv \frac{F(x,T) - M_{(N,n)}}{2\pi LT^2 n} = \frac{1-x}{xt^2} - \lambda_o\, K\,(xt)^{p-1} + x - 1\,, \tag{2.22}$$

where $K$ is a positive constant of $\mathcal{O}(1)$, and we have used the exact expression for the binding energy of $n - n'$ F1-strings in the NCOS limit:

$$\begin{aligned} M_x &\equiv \lim\nolimits_{\mathrm{NCOS}} \left[M_{(N,n')} + M_{(0,n-n')} - M_{(N,n)}\right] \\ &= \frac{N^2 L V_\perp^2}{4\pi(4\pi^2\alpha'_\perp)^{p-1}\, g_s^2 \alpha'} \left(\frac{1}{n'} - \frac{1}{n}\right) = 2\pi\, L T_H^2\, n\, \frac{1-x}{x}\,. \end{aligned} \tag{2.23}$$

In order to determine the equilibrium value of $x$ we minimize (2.22) with respect to the ionization fraction $x$ at fixed temperature $T > T_H$, i.e. we seek local minima, characterized by $\partial_x f(x,t) = 0$:

$$(p-1)K\,(\lambda_o t)\,(xt)^p - (xt)^2 + 1 = 0\,. \tag{2.24}$$

This is equivalent to the equality of chemical potentials:

$$\mu_{\mathrm{bs}} - \mu_{\mathrm{F1}} = \frac{\partial F_{\mathrm{bs}}}{\partial n'} - \frac{\partial F_{\mathrm{F1}}}{\partial n'} = 0 \tag{2.25}$$

that expresses canonical equilibrium at a fixed temperature.

Using (2.24) we can solve for the leading coupling correction to the ionization fraction $x(t)$. First, we need to assume that the bound state remains weakly coupled at $t \gg 1$. Since $\lambda_o(t) \approx \lambda_o \cdot t$ to leading order, this condition allows us to seek the large $t$ solution of (2.24) by perturbing the free solution. One finds

$$x(t) = \frac{1}{t} + (p-1)K\,\lambda_o + \mathcal{O}(\lambda_o^2)\,, \tag{2.26}$$

It is interesting that this is a positive shift, in agreement with the general idea that the volume-shrinking effect tends to inhibit the ionization.

Plugging $x = 1$ in (2.24) we find the free-energy gap in ionizing the *first* F1-string:

$$\Delta F_{\mathrm{ion}} = (\mu_{\mathrm{F1}} - \mu_{\mathrm{bs}})_{x=1} = 2\pi\, L T_H^2 \left(1 - t^2 + (p-1)\, K\, \lambda_o\, t^{p+1}\right), \tag{2.27}$$

which shows the temperature barrier for ionization, proportional to $\lambda_o T^{p+1}$. The critical ionization temperature corresponds to the vanishing of this free-energy gap. Close to the Hagedorn temperature $T \approx T_H$, one finds

$$T_{\mathrm{critical}} = T_H \left(1 + \frac{1}{2}(p-1)K\lambda_o + \mathcal{O}(\lambda_o^2)\right). \tag{2.28}$$

Again, this is a *positive* shift, so that indeed the volume effect inhibits the ionization to some extent. One may wonder if this positive shift of the critical temperature is not bringing in the physics of long strings. However, the effect is only significant for $\lambda_o \approx 1$. Within the weak-coupling regime, we expect that the shift is superseded by the matching uncertainties involved in the parametrization (2.20).

An expression equivalent to (2.27) was derived for $p = 3$ using the supergravity description in [22]. In the last section of the paper we extend (2.27) to the NCOS supergravity regime for general values of $p$. The weak-coupling calculation leading to the thermal barrier (2.27) also applies to the ionization of D1-strings in a $(\mathrm{D3}, \mathrm{D1})$ bound state, in the low-energy limit that defines noncommutative $\mathcal{N} = 4$ Super Yang-Mills theory [31, 24]. In fact, it is the $S$-dual of our calculation. The effective open-string metric of the noncommutative field theory is $G_{ij} = \delta_{ij}\, \alpha'_e/\alpha'_\perp$, by the standard action of $S$-duality on the formulas of section 1.1, which results in the same volume-shrinking effect. The rest of the ingredients are also present: the free energies on the bound state and the ejected D1-strings are saturated by massless fields and the gap for D1-string ionization is the $S$-dual of the gap for F1-string ionization. The final result is an expression for thermal free energy barrier valid for $g^2 N \ll 1$, with $g$ the Yang-Mills coupling:

$$(\Delta F)_{\rm ion} = -2\pi L T^2 + \frac{\pi L}{\theta g^2} + 4\pi C N L \theta T^4 \,, \tag{2.29}$$

where $\theta$ is the noncommutativity parameter of the Yang-Mills theory and the constant

$$C \equiv \frac{30\, \mathrm{Vol}(\mathbf{S}^2)}{(2\pi)^2}\, \zeta(4)\,.$$

As expected, the result is the $S$-dual of (2.27) for $p = 3$ and coincides with the supergravity calculation of ref. [22]. It shows that there is no D1-string ionization in the weakly-coupled noncommutative Yang-Mills theory at finite temperature.

**F1-string dominance versus recombination.** The most important consequence of the effective volume is to render the entropy of the bound-state effectively two-dimensional deep inside the ionization phase. Assuming that ionization takes place at $T \gg T_H$, so that $x(t) \approx 1/t$, we have

$$S_{\rm bs} = N^2 C V_\perp L T^p x^{p-1} \sim N^2 V_\perp L T_H^{p-1} T\,. \tag{2.30}$$

Since the entropy in ionized material is

$$S_{\rm F1} = 4\pi\,(n - n')LT \approx 4\pi L n (T - T_H) = 2^{\frac{p-4}{2}} \pi^2 N^2 V_\perp L T_H^{p-1} \frac{T - T_H}{\lambda_o}\,, \tag{2.31}$$

we get dominance of F1-component for sufficiently weak coupling:

$$\lambda_o \lesssim 1 - \frac{T_H}{T}\,, \tag{2.32}$$

or $\lambda_o \lesssim 1$ for $t \gg 1$. Notice that this is the balance line of the two-dimensional system. Therefore, we find that the entropy becomes F1-dominated roughly at the same temperature, independently of the dimension of the NCOS, showing rather sharply how all NCOS theories are effectively two-dimensional in the ionization regime. We stress that the volume-shrinking effect is crucial in obtaining this universality of the onset of F1-string domination.

This result suggests that the supergravity matching of the ionized bound state is essentially governed by the $1+1$ dimensional theory on the world-volume of the F1-strings, i.e. by the matrix-string phase [32]. Thus, one can anticipate that the supergravity matching line is

$$\lambda_o \sim \frac{1}{t^2}\,, \tag{2.33}$$

universally, for all values of $p < 5$.

The previous considerations are based on the equilibrium configuration $x(t) \approx 1/t$ for $t \gg 1$. Notice, however, that the free-energy gap for emission of the first F1-string (2.27) can also vanish at $t \gg 1$, i.e. there is a second solution of (2.24) at $x = 1$ of the form

$$\lambda_o \sim \frac{1}{t^{p-1}}\,. \tag{2.34}$$

Thus, it is possible that the ionization process is reversed and 'recombination' of the F1-strings occurs precisely for $\lambda_o\, t^{p-1} \gg 1$. Physically, we can understand this critical line by the matching of the entropy in massless fields on the un-ionized bound state, given by formula (2.30) with $x = 1$, and the entropy in ionized F1-strings, given by (2.31).

For $p \geq 3$, such recombination can take place with a small effective coupling on the bound state $\lambda_o(t) = \lambda_o t \ll 1$. Actually, the weak-coupling analysis is only valid below the supergravity correspondence line $\lambda_o t^2 \sim 1$, so that for $p = 3$ the issue must be studied within the supergravity description [22]. On the other hand, for $p = 4$ the large hierarchy of couplings $1/t^3 \ll \lambda_o \ll 1/t^2 \ll 1$ appears to correspond to a weakly-coupled 'recombined' NCOS bound state.

To be more precise, we can study the global shape of the function $f(x,t)$ in the interval $x \in [0,1]$. Notice that, at $x = 1$, the derivative $\partial_x f(1,t)$ changes sign for $\lambda_o \sim t^{1-p}$ and is large and negative for $\lambda_o \gg t^{1-p}$. The value of $f(1,t) \approx -\lambda_o\, t^{p-1}$ is smaller than the value at the local minimum, $f_{\rm min} \sim -1$, precisely when $\lambda_o\, t^{p-1} \gg 1$. Thus, the ionization fraction at the local minimum $x(t) \approx 1/t$ is only metastable for $\lambda_o\, t^{p-1} \gg 1$, as expected. For $p = 4$ the absolute minimum of the free energy is at $x = 1$ in the region $1/t^3 \ll \lambda_o \ll 1/t^2$, clearly inside the weak-coupling domain.

For $\lambda_o\, t^{p-1} \sim 1$ the system described by these thermodynamic functions undergoes a first-order phase transition whereby islands of the original bound state with $x = 1$ nucleate and grow inside the medium at $x(t) \approx 1/t$. During the nucleation process, the free energy as a function of $x$ is given by the convex envolvent of the function appearing in (2.22). From the macroscopic point of view, working at fixed values of bulk parameters, the nucleation process is described by the emergence of inhomogeneities of the electric field. Relating $\mathcal{E}$ and $n$ via eq. (1.14), the recombination is nothing but the growth of inhomogeneities with maximal electric field in a medium with electric field appropriate to the F1-string density $nx$.

The possibility of weak-coupling recombination for $p = 4$ raises a puzzle in relation to the supergravity matching. In the intermediate regime $(T_H/T)^3 \ll \lambda_o \ll (T_H/T)^2$ the entropy is dominated by five-dimensional massless fields, which give much too large an entropy at the correspondence line. As we will see in the next section, the analysis of the supergravity solutions strongly suggest that the correspondence line is given by (2.33) and that the system must be dominated by ionized F1-strings at that temperature. Hence, if recombination takes place, the supergravity correspondence line coincides with a first-order phase transition with enormous latent heat $\Delta Q \sim T(S_{\rm bs} - S_{\rm F1}) \sim TS_{\rm bs} \sim E_H/\lambda_o^{5/2}$. In such a situation, the correspondence principle itself loses much of its predictive power. Of course, it is possible that our estimates of the thermodynamic functions are wrong due to some unknown infrared divergences that are specially strong for $d = 5$ (see [48] for related phenomena in a slightly different context).

Another possibility is that our assumptions on the effective quenching of long open strings are wrong. In particular, the ansatz for the free energy in (2.20) was only justified in the close vicinity of the effective Hagedorn temperature of the bound state. The peculiar features of the function $f(x)$ that are responsible for recombination at $x = 1$ become effective for temperatures much larger than the equivalent Hagedorn temperature of the bound state with $x = 1$. Thus, the assumption that only massless fields enter the dynamics may be wrong very far from equilibrium. In the next section we show that long open strings effectively wash out the recombination first-order phase transition. However, such an scenario fails the test of the supergravity matching for *any* value of $p$. Thus, we are left with a genuine puzzle for $p = 4$. All this being said, any outcome may give some hint at the induced behaviour of the OM system.

### 2.3 Microcanonical analysis: no F1-string ionization

Having discussed the canonical ensemble along the lines of ref. [21] we now turn to the microcanonical analysis. The canonical ensemble was *assumed* to be dominated by massless degrees of freedom, and this assumption yields a consistent picture with positive specific heat. Therefore, one may expect that the microcanonical treatment should simply vindicate this picture. On the other hand, if NCOS systems bear some similarity to standard D$p$-branes with $p < 5$, long open strings should dominate the microcanonical ensemble because they have the highest density of states. In this section we confirm this dicotomy.

Our main hypothesis is that the coupling $\lambda_o$ is sufficiently small, so that the system is well approximated by non-interacting components: massless excitations on the ejected F1-strings and string excitations on the (D$p$, F1) bound state. In addition, the string excitations on the bound state are divided into massless modes and long open strings. The total energy is

$$E = E_s + E_g + E_f + M_x \,, \tag{2.35}$$

where $E_s$ denotes the energy in long open strings attached to the bound state, $E_g$ refers to the energy in the form of massless excitations on the $(p+1)$-dimensional world-volume of the bound state, $E_f$ is the energy in collective modes of the ejected fundamental strings, and $M_x$ is the energy gap for ejection of these F1-strings, as in (2.23).

We approximate the entropy as a non-interacting mixture

$$S(E) = S_s(E_s) + S_g(E_g) + S_f(E_f) \,. \tag{2.36}$$

Here, $S_s$ is the entropy in long strings, for which we assume the Hagedorn form

$$S_s = c_s \sqrt{8\pi^2 \alpha'_e}\, E_s = c_s\, x\, \frac{E_s}{T_H} \,, \tag{2.37}$$

where $c_s = 1 + \mathcal{O}(\lambda_o)$. In the following we absorb $c_s$ into the definition of $T_H$. Next, the entropy in massless fields on the D$p$-brane is

$$S_g = N^2 C' V_{\text{eff}} \left( \frac{E_g}{N^2 C' V_{\text{eff}}} \right)^{\frac{p}{p+1}} = x^{\frac{p-1}{p+1}} N^2 C' V \left( \frac{E_g}{N^2 C' V} \right)^{\frac{p}{p+1}} \,, \tag{2.38}$$

with $C'$ an $\mathcal{O}(1)$ constant. Finally, the entropy in collective modes of F1-strings is

$$S_f = \sqrt{8\pi (n - n') L E_f} = \sqrt{8\pi n (1-x) L E_f} \,. \tag{2.39}$$

With these ingredients we are ready to study the balance. Let us start with the original bound state with $x = 1$ at low energies. The energy gap for ionizing the first F1-string is of order

$$E_{\text{ion}} = 2\pi L T_H^2 \,. \tag{2.40}$$

The energy for exciting any long open strings on the bound state is of order $T_H$. Since we essentially neglect finite-size effects in this paper, we have $L T_H \gg 1$ and thus $E_{\text{ion}} \gg T_H$, i.e. when the ionization becomes possible, there is enough energy to excite long strings in the system and all channels of (2.35) are open.

The origin of the threshold $E_{\text{ion}}$ is the discreteness of the ionization fraction $x = n'/n$, with step $1/n$. Thus, we consider $E \gg E_{\text{ion}}$ so that we can approximate $x$ by a continuous variable. Then, for a given total energy $E$, there is a minimum value of $x$ compatible with the splitting (2.35). It corresponds to using up all the available energy in ionizing a maximal number of F1-strings that remain at zero temperature, i.e. $E = M_x$. Using the explicit form of $M_x$ in (2.23) we find

$$x_m = \frac{n\, E_{\text{ion}}}{E + n\, E_{\text{ion}}} \,. \tag{2.41}$$



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Let us now assume some fixed ionization fraction $x > x_m$ and consider an energy large enough to have all channels in thermal equilibrium. If the total entropy has a local maximum, it corresponds to the equilibrium condition that the microcanonical temperatures

$$\frac{1}{T_i} = \frac{\partial S_i}{\partial E_i}$$

are equal for all components. In particular, equal to the temperature of long strings, given by $T_s = T_H/x$. From the equations $T_g = T_f = T_s$ we obtain the values of $E_g$ and $E_f$ as a function of $x$:

$$E_g = \frac{E_{gc}}{x^2}\,, \qquad E_f = E_{fc}\,\frac{1-x}{x^2}\,, \tag{2.42}$$

where the 'critical energies' are given by

$$E_{gc} \equiv \left(\frac{p}{p+1}\right)^{p+1} N^2 C' V T_H^{p+1} \approx E_H, \qquad E_{fc} \equiv 2\pi n L T_H^2 \approx \frac{E_H}{\lambda_o} \gg E_H\,, \tag{2.43}$$

where we have used the expression for $n$ in terms of the 't Hooft coupling and the weak-coupling condition $\lambda_o \ll 1$. Thus, the energy in massless gases, either on the D$p$-brane world-volume or on the F1-strings, attains a fixed value for a given ionization fraction. This means that for sufficiently large total energy $E$, at fixed $x$, most of the energy is in long strings

$$E_s = E - E_f - E_g - M_x\,, \tag{2.44}$$

which grows linearly with $E$. This is the expected behaviour, and it should not be significantly affected by the addition of logarithmic corrections to the Hagedorn spectrum.

On the other hand, the energy stored in anything but long strings,

$$E - E_s = \frac{E_{gc}}{x^2} + E_{fc}\frac{1-x^2}{x^2}\,, \tag{2.45}$$

is a monotonically decreasing function of $x$. Thus, for a fixed total energy $E$, there is a minimum value of the ionization fraction that is compatible with having excited long open strings. We find this value by setting $E_s = 0$:

$$x_s = \sqrt{\frac{E_{gc} + E_{fc}}{E + E_{fc}}} \approx \sqrt{x_m}\,. \tag{2.46}$$

Since $x_s < 1$ only for $E > E_H$, we see that below this threshold the long strings are absent form the thermodynamic balance for all values of $x$.

More generally, this is also true in the window $x_m < x < x_s$: the effective Hagedorn temperature of the bound state is larger than the actual microcanonical temperature, and the long-string channel is closed, so that the balance in this region is between massless Yang-Mills fields on the bound state and the ejected F1-strings, i.e. exactly the system treated canonically in the previous subsection.

Introducing the temperature as a function of $x$ by the equation

$$E = E(x, T(x)) \,, \tag{2.47}$$

with the energy function

$$E(x,T) = \left(\frac{p}{p+1}\right)^{p+1} N^2 C' V T^{p+1} x^{p-1} + 2\pi n L T^2 (1-x) + 2\pi n L T_H^2 \, \frac{1-x}{x} \,, \tag{2.48}$$

the boundary conditions are that the system is at the onset of long-string excitation for $x = x_s$, and totally dominated by binding energy for $x = x_m$, i.e. $T(x_s) = T_H/x_s$ and $T(x_m) = 0$. Since long open strings are absent for $x_m < x < x_s$, we have $T(x) < T_H/x$ throughout this range.

The problem of maximizing the entropy function

$$S(x,E) \equiv S(x,T(x)) = \left(\frac{p}{p+1}\right)^{p} N^2 C' V T^{p} x^{p-1} + 4\pi n L T (1-x) \,, \tag{2.49}$$

with fixed total energy is related to the problem solved in the previous section, where we discussed the minimization of the free energy $F(x,T)$ at fixed temperature. A simple manipulation of Legendre transforms shows that

$$\frac{dS(x)}{dx} = -\frac{1}{T} \partial_x F(x,T) \,. \tag{2.50}$$

Therefore, the graph of $S(x)$ is qualitatively similar to the graph of $F(x,T)$, when drawn upside down. In particular, the local maximum of $S(x)$ is at $x = x_s$. The recombination effect discussed before is also visible. One finds that for sufficiently strong coupling

$$\lambda_o > \left(\frac{E_H}{E}\right)^{\frac{p-1}{p+1}} \tag{2.51}$$

the entropy function develops a global maximum at $x = 1$. However, in the micro-canonical ensemble with long strings, it is clear that this analysis only applies to the interval $x_m < x < x_s$. Thus, the recombination appears as a completely spurious phenomenon, superseeded by the emergence of long strings in the region $x > x_s$.

We can summarize the physics of the region $x_m < x < x_s$ by starting at the lower end, with the system at zero temperature and maximal ionization compatible with the given total energy. In order to increase the entropy while keeping the total energy constant, we must increase $x$ and excite massless fields on the bound state and the F1-strings. This process continues until the long strings can be excited on the bound state at $x = x_s$.

It remains now to consider the region $x > x_s$ where all components are active. The most probable configurations at fixed $x$ give entropies:

$$
\begin{aligned}
S_g(x) &= S_{gc}\,\frac{1}{x}\,, \\
S_f(x) &= S_{fc}\,\frac{1-x}{x}\,, \\
S_s(x) &= \frac{x}{T_H}\left(E - E_{gc}\,\frac{1}{x^2} - E_{fc}\,\frac{1-x}{x^2} - M_x\right),
\end{aligned} \tag{2.52}
$$

where the critical entropies are given by

$$
S_{gc} \equiv \left(\frac{p}{p+1}\right)^p N^2 C' V T_H^p\,, \qquad S_{fc} \equiv 4\pi n L T_H\,. \tag{2.53}
$$

Adding all the terms up we find

$$
S(x) = x\left(\frac{E+E_{fc}}{T_H}\right) - S_{fc} - \frac{1}{x}\left(\frac{F_{gc}+F_{fc}}{T_H}\right), \tag{2.54}
$$

where we have defined the critical free energies in the obvious fashion $F_c = E_c - T_H S_c$.

In the interval $0 < x < 1$, the function $S(x)$ is monotonically increasing around $x = 1$ (notice that both $F_{fc}$ and $F_{gc}$ are strictly negative). It attains a minimum at

$$
x_- = \sqrt{\frac{|F_{gc}+F_{fc}|}{E+E_{fc}}}\,, \tag{2.55}
$$

and it grows without bound towards $x = 0$. By direct inspection one finds that

$$
x_- \lesssim x_s\,, \tag{2.56}
$$

which means that $S(x)$ is monotonically increasing in the interval $x_s < x < 1$. Therefore, the system continues gaining entropy by increasing $x$ beyond $x_s$. The global maximum lies at $x = 1$.

This analysis shows that the F1-ionization does not take place in the weakly-coupled microcanonical ensemble once we allow the long open strings as effective degrees of freedom. The result is smooth in the dimension of the D$p$-brane and only depends on the weak-coupling assumptions. According to this result, the typical string configurations will be those of long strings.

**A possible interpretation** The lack of F1-ionization in the microcanonical analysis is intuitively natural. Even if the system can dissociate and remain with possitive specific heat, the entropy gain in exciting the long open strings is so large that this is by far the most probable configuration at *finite* and large total energy.

As mentioned in the introduction, it is not uncommon that phases with Hagedorn behaviour and negative specific heat are simply not seen in the canonical ensemble.



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A good example of such a behaviour is the thermodynamics of $\mathcal{N}=4$ SYM theory on $S^3$ with radius $R$, as obtained via the AdS/CFT correspondence. In the canonical ensemble at large $N$, the vacuum-dominated thermodynamics jumps to a plasma phase at temperatures of $\mathcal{O}(1/R)$. In terms of the AdS supergravity picture, this corresponds to the Hawking-Page transition, i.e. the formation of an AdS black hole with positive specific heat and horizon radius comparable to the curvature radius of the AdS space [33, 17].

On the other hand, the microcanonical analysis reveals two more phases that appear as clear transients for large values of the 't Hooft coupling $\lambda \gg 1$, and are associated to finite-size effects in the gauge theory [34, 35, 36]. In the AdS picture they are related to the emergence of long closed strings at energy densities larger than the type IIB string scale. A Hagedorn spectrum of long closed strings with entropy

$$S_{\text{Hagedorn}} \sim \ell_s E \sim \frac{RE}{\lambda^{1/4}}$$

matches at the lower energy end, of order $ER \sim \lambda^{5/2}$, to a massless graviton gas with entropy

$$S_{\text{gas}} \sim (RE)^{9/10}\,.$$

At higher energies of order $ER \sim N^2\lambda^{-7/4}$ the long strings match to ten-dimensional Schwarzschild black holes, fully localized in the $AdS_5 \times S^5$ background, with entropy

$$S_{\text{Schwarzschild}} \sim N^{-2/7}(RE)^{8/7}\,.$$

Finally, the small black holes delocalize in the $S^5$ and merge with the large AdS black hole at energies of order $ER \sim N^2$.

Thus, the long-string phase and the Schwarzschild black-hole phase of $AdS_5 \times S^5$ are bounded transients with negative specific heat that are not seen at all in the canonical ensemble, which jumps directly from the graviton gas to the large AdS black hole. It could seem natural to suspect that an analogous phenomenon would explain the long-string phase at the NCOS Hagedorn temperature $T_H$ that is borne out by the microcanonical analysis in this section, i.e. if the long NCOS strings are true effective degrees of freedom competing for the entropy at very high energy densities, then there should be appropriate black-hole phases to match them at strong coupling.

The likelihood of having such transients in NCOS systems is *a priori* small. First order phase transitions identified in a canonical analysis manifest themselves as transient behaviour in the microcanonical analysis of the same system. The latent heat inherent in the first-order transition is correlated with the energy range over which the transient structure emerges in the microcanonical ensemble and viceversa; the transient behaviour in the microcanonical ensemble implies a first-order transition in the canonical system. Thus, the existence of transient long-string phases would suggest a first-order phase transition in the NCOS system. However, ref [21] gives strong arguments that the phase transition in the NCOS is essentially of second order. It is thus unlikely that the phase of long strings in this section can find its place in a transient. All this brings us to the study of the supergravity phase diagrams for these systems, as an arbitror on the fate of Hagedorn NCOS strings. If the corresponding black-hole metrics are not found, we would conclude that the long open strings are not appropriate degrees of freedom at high energy densities.

## 3. Phase diagrams

In this section we survey the phase diagrams of NCOS systems, constructed using the qualitative methods of [15]. Our main interest is to study the nature of the physics around the Hagedorn transition of the NCOS theories. However, in some cases it is instructive to look at the complete phase diagram including nonperturbative $S$-duality transitions. One such case is the four-dimensional NCOS theory, related by $S$-duality to a noncommutative Yang-Mills theory. We will use this system as a detailed example to carry the discussion through, and quote at the end the appropriate generalization to other dimensionalities. Various pieces of the discussion have appeared already in the literature cf. [19, 20, 22]

We show that supergravity considerations essentially leave no room for a manifest NCOS Hagedorn regime at weak coupling, in the sense of long-string domination with characteristic temperature $T_H \sim 1/\sqrt{\alpha'_e}$. In order to keep in sight all possible transient phases, we study the gravitational thermodynamics both in the canonical and microcanonical ensembles, and we also consider 'near NCOS' limits, in an attemp to make contact with the results known for pure D$p$-branes.

### 3.1 The phase diagram of $3+1$ dimensional NCOS

The four-dimensional theory arises as the NCOS limit of a bound state $(\mathrm{D3}_N, \mathrm{F1}_n)$. Under type IIB $S$-duality, the bound state transforms into $(\mathrm{D3}_N, \mathrm{D1}_n)$ and the NCOS limit is mapped to the low-energy limit that defines $\mathrm{U}(N)$ SYM with space-space noncommutativity (NCYM). The $S$-duality relations at the level of closed-string parameters: $\tilde{g}_s = 1/g_s$ and $\alpha'_\perp = g_s \alpha'$ induce the corresponding mapping of open-string parameters.

We choose $\alpha'_\perp = \alpha'_e$ throughout this section, so that the open-string metrics on both sides of the duality are given by the Minkowski metric. Then, the Yang-Mills coupling of the NCYM theory is $g^2 = 2\pi/G_o^2$. In terms of the 't Hooft coupling $\lambda \equiv g^2 N$:

$$\lambda = \frac{(2\pi N)^2}{\lambda_o} \,. \tag{3.1}$$

The noncommutativity parameter $\theta$ of the NCYM theory is given by

$$\theta = \frac{NV_\perp}{2\pi n} = 2\pi\alpha'_\perp G_o^2 \,. \tag{3.2}$$

A useful relation valid in the NCOS limit is [37]:

$$2\pi\theta_e = g^2\theta \,, \tag{3.3}$$

explicitly relating the noncommutativity parameters on both sides of the $S$-duality.

Thus, in drawing the phase diagram of the four-dimensional NCOS theory, the candidate phases are the U($N$) NCYM theory with space-space noncommutativity, its supergravity dual (with and without a nearly extremal black hole) and the respective $S$-dual configurations. That is the open-string theory which is time-space noncommutative in the NCOS limit as well as the $S$-dual gravitational configurations. We will draw the boundaries separating the different phases. A special property of the four-dimensional case is that all of the supergravity and weakly-coupled YM 'phases' have a common functional form for the entropy as a function of temperature or energy. For high enough energies it is that of a four-dimensional field theory of massless particles. Therefore, it will be of no wonder that there is no room for a phase in which the entropy is stringy, i.e. linear in energy. This will indeed turn out to be the case. This dramatic failure at a naive matching of Hagedorn thermodynamics motivates in part our choice of the four-dimensional case as the specific example to carry out the discussion.

The phase diagram is expressed in a two-dimensional plane whose coordinates are a 't Hooft coupling and a running energy, $u$, respectively. It will be useful to describe the system once in terms of the U($N$) SYM 't Hooft coupling $\lambda \equiv g^2 N$ and once in terms of the (stringy) 't Hooft coupling of the NCOS theory, $\lambda_o = 2\pi G_o^2 N$. We do not discuss finite-size effects in this paper, so that we only consider the leading thermodynamic behaviour in the limit of large world-volumes.

We start from the region of weak coupling of the NCYM theory, parametrized by the infrared 't Hooft coupling $\lambda$ and the (perturbative) noncommutativity length scale $\sqrt{\theta}$. For very small $\lambda$ and very small energies the system is well described as an ordinary U($N$) SYM gauge theory, as the system is not yet sensitive to the its non-commuting character. This lasts as long as the energy $u$ is smaller than $1/\sqrt{\theta}$. Above this energy, for the same small value of the coupling, the system should be sensitive to the noncommutativity of space.

As the 't Hooft coupling increases to $\lambda \approx 1$, the perturbative gauge-theory picture starts to crack. At large enough values of $N$ the supergravity description becomes the appropriate one. The large-$N$ master field of the theory can be described via the AdS/CFT correspondence in terms of type IIB strings on the background [38, 39]:

$$\frac{1}{R^2}(ds^2)_{\rm NCYM} = u^2\left(-dt^2 + dy^2 + f(u)d\mathbf{x}^2\right) + \frac{du^2}{u^2} + d\Omega_5^2 \,, \tag{3.4}$$

$$B = \frac{1}{\theta}(a_\theta u)^4 f(u), \qquad e^{2\tilde{\phi}} = \left(\frac{\lambda}{2\pi N}\right)^2 f(u)\,, \tag{3.5}$$

where the nontrivial profile function is

$$f(u) = \frac{1}{1+(a_\theta u)^4}\,, \tag{3.6}$$

and the curvature radius of the $AdS_5 \times S^5$ geometry at the infrared $u \to 0$ is $R^4 = 4\pi \tilde{G}_s N(\tilde{\alpha}')^2 = 2\lambda(\tilde{\alpha}')^2$. The coordinates are chosen so that $u$ measures the field-theory energy scale, i.e. a black-hole solution has a horizon at $u_0 \sim T$, with $T$ the Hawking temperature (the physical temperature of the gauge-theory dual). On the supergravity side the important scale from the physical point of view is the noncommutativity length, $a_\theta$, which is related to the noncommutativity parameter $\theta$ through a certain dressing by powers of the 't Hooft coupling:

$$(a_\theta)^4 = \lambda \frac{\theta^2}{2\pi^2}\,. \tag{3.7}$$

The onset of noncommutative effects is at the energy scale $u \sim 1/\sqrt{\theta}$ for weak coupling. In the supergravity background, effects of the magnetic field become important around the line $ua_\theta \sim 1$, or

$$\lambda \sim \frac{1}{\theta^2 u^4}\,. \tag{3.8}$$

The curvature of the supergravity solution in string units turns out to be of $\mathcal{O}(\alpha'/R^2)$ times a bounded factor depending on $(a_\theta u)$ and of $\mathcal{O}(1)$. Thus the master-field description in terms of supergravity ceases to be reliable at the Horowitz-Polchinski (HP) transition, that point/line in parameter space where the curvature becomes of $\mathcal{O}(1)$ in string units. This actually occurs for $\lambda \sim 1$, coinciding with the line which serves as the boundary for the onset of the breakdown of the perturbative picture. At $\lambda < 1$ we must use perturbative techniques in the analysis, whereas for $\lambda > 1$ we can use the metric (3.4).

The above supergravity description is valid as long as the closed-string coupling itself is small. At sufficiently large coupling, $\lambda \sim N$ the local value of the closed-string coupling constant is of order one, $\exp(\tilde{\phi}) = \mathcal{O}(1)$. This defines a crossover line

$$\lambda \sim N \qquad \text{for } a_\theta u \ll 1\,, \qquad \text{and} \qquad \lambda \sim N^2\theta^2 u^4\,, \qquad \text{for } a_\theta u \gg 1\,, \tag{3.9}$$

to a description based on the $S$-dual background with $\tilde{\phi} \to \phi = -\tilde{\phi}$ and metric

$$\frac{1}{R_o^2}(ds^2)_{\rm NCOS} = \frac{1}{\sqrt{f(u)}}\left[u^2\left(-dt^2 + dy^2 + f(u)d\mathbf{x}^2\right) + \frac{du^2}{u^2} + d\Omega_5^2\right] \tag{3.10}$$

with $R_o^4 = 4\pi G_o^2 N(\alpha'_e)^2$. The deep infrared of this metric describes again $AdS_5 \times S^5$ with radius of curvature $R_o$. This is in agreement with the expected $S$-duality of low-energy U($N$) SYM theory. Incidentally, we note that $R_o = a_\theta$. This embodies the fact that noncommutative effects of the NCOS theory are tied to the string scale $\sqrt{\alpha'_e}$ [40, 1, 41]. On the other hand, in the high-energy regime $uR_o \gg 1$, where the noncommutativity effects are most evident, the metric (3.10) is asymptotic to that of smeared F1-strings [19].

Roughly speaking, we can characterize the metric (3.10) of the (D3, F1) bound state as dominated by the D3 component in the low-energy regime, and dominated by the F1 component in the high-energy regime. The crossover is at energies of $\mathcal{O}(1/R_o)$.

Our main interest is the matching of the metric (3.10) to perturbative NCOS at a sufficiently weak NCOS coupling. The corresponding HP transition could be considered '$S$-dual' of the already discussed HP transition to perturbative NCYM. In discussing the $S$-dual HP transition, we remark that the $S$-dual metric (3.10) is globally conformally related to (3.4):

$$(ds^2)_{\rm NCOS} = \frac{\alpha'_e}{\tilde{\alpha}'} e^{-\tilde{\phi}} (ds^2)_{\rm NCYM} \,. \tag{3.11}$$

Since the Ricci curvature of the NCYM metric is of order $\lambda^{-1/2}$ in string units, the HP transition to the weak-coupling NCOS is given by (under conformal transformations the Ricci tensor is 'contravariant'):

$$\frac{e^{\tilde{\phi}}}{\sqrt{\lambda}} \sim 1 \,, \qquad \text{or} \qquad \lambda \sim N^2 \left(1 + (a_\theta u)^4\right) . \tag{3.12}$$

This gives the expected $\lambda \sim N^2$ at $ua_\theta \ll 1$, or $\lambda_o \sim 1$ in terms of the NCOS 't Hooft coupling. On the other hand, it gives a condition asymptotically independent of the NCYM coupling as $ua_\theta \gg 1$, namely

$$u \sim \frac{1}{\sqrt{\theta N}} \,. \tag{3.13}$$

This means that the extreme ultraviolet regime in the NCOS region is well described by supergravity. The behaviour is to be contrasted with that at weak coupling, where the transition between the supergravity and the gauge pictures occurs at essentially the same coupling for all energies. At the large coupling end the transition line is not only energy-dependent, for energies larger than $1/\sqrt{\theta N}$ there is no transition. The supergravity picture continues to be appropriate no matter how large the coupling is. At lower energies a transition occurs back to a field theory. For very low energies this field theory is the one $S$-dual to the U($N$) gauge theory of weak coupling.

In NCOS variables, the HP transition line (3.13) is given by

$$\lambda_o \sim \frac{1}{\theta_e u^2} \,. \tag{3.14}$$

On the other hand, the noncommutative crossover in the supergravity regime, $ua_\theta = uR_o \sim 1$, is expressed in NCOS variables as

$$\lambda_o \sim \frac{1}{\theta_e^2 u^4} \,. \tag{3.15}$$

This curve intersects the NCOS correspondence line (3.14) at the energy scale $u \sim 1/\sqrt{\theta_e} = \sqrt{N/\lambda\theta}$. This is the energy scale at which the NCOS theory should start showing stringy features. We will see shortly that this energy is much lower than that at which the supergravity picture takes over, but it actually coincides with the expected Hagedorn temperature of the NCOS system: $T_H \sim 1/\sqrt{\theta_e}$. It is the exact nature of this supposedly Hagedorn temperature that we are exploring.

**No place for a truly Hagedorn phase.** At finite temperature the previous metrics get the usual black-hole generalization with horizon radius $r_0 = u_0/R^2$:

$$dt^2 \to h dt^2 \,, \qquad dr^2 \to \frac{dr^2}{h} \,, \qquad h = 1 - \left(\frac{r_0}{r}\right)^4 \,,$$

and the previous phase diagram becomes a thermodynamic phase diagram with $u_0 \sim T$. This is correct as long as the specific heat is positive. Otherwise we would have had to reinterpret $u_0$ in terms of the total energy of the system. Such a problem would occur if the NCOS underwent a Hagedorn transition to a regime dominated by *long* open strings, with entropy:

$$S_{\rm Hag} \sim \sqrt{\theta_e} E \,, \tag{3.16}$$

and approximately constant Hagedorn temperature $T_H \sim 1/\sqrt{\theta_e}$. This would have to occur precisely at the onset of noncommutative effects in the NCOS, i.e. along the line

$$T \sim \frac{1}{\sqrt{\theta_e}} \sim T_H \,. \tag{3.17}$$

Therefore, in the full NCOS region with $\lambda_o \ll 1$ and

$$T_H \ll T \ll \frac{T_H}{\sqrt{\lambda_o}} \,, \tag{3.18}$$

we should expect a Hagedorn phase of long-string domination, which would be constrained to match to supergravity at the upper limit. The enormous disparity of the upper and lower temperature limits in (3.18) when $\lambda_o \ll 1$ shows that the physics of this region of parameter space cannot be described in terms of a standard Hagedorn regime, since long-string dominance (3.16) always leads to approximately constant temperature.

Another way of phrasing the problem is to recognize that the thermodynamics is field-theoretical throughout all the rest of the phase diagram. For example the entropy in the large-$N$ approximation is of the form (we assume $LT, LV_\perp T^3 \gg 1$)

$$S \sim N^2 V_\perp L T^3 \sim N^2 V_\perp L \left(\frac{E}{N^2 V_\perp L}\right)^{3/4}, \tag{3.19}$$

either in perturbative SYM phases or in the supergravity phases. This follows from the conformality of the low-energy limits in both perturbative SYM and supergravity descritions, together with the fact that noncommutative effects in the thermodynamic functions do not show up at the planar $\mathcal{O}(N^2)$ level [39, 42, 43, 44]. On the other hand, the matching of this entropy law to the Hagedorn one (3.16) is already determined to be at the noncommutativity line (3.17), which describes the transition from the ordinary low energy U($N$) SYM to NCOS. Although the estimates are rather crude and could be off by numerical coefficients, there is no evidence that they should involve functions of $\lambda_o$.

Thus it does not seem possible to have a 'truly Hagedorn' behaviour in an NCOS regime matching the supergravity part. By 'truly Hagedorn' we mean one in which the linear dependence of the entropy on the energy is activated and manifest. It could in principle be that somewhere deep inside the NCOS region there is a 'Hagedorn enclave', but we do not know what would be its surrounding physics. The 'Hagedorn crisis' is then the failure of the entropy matching at the upper limit of (3.18).

In hindsight, the 'Hagedorn crisis' described here was to be expected, in view of the thermodynamic properties of the supergravity description. Namely the corresponding black branes have positive specific heat. On the other hand, all known examples of HP transition between a Hagedorn phase of long (open or closed) strings match to black geometries of the Schwarzschild type, and in particular with systems which have negative specific heat. Thus, it would actually be odd from the point of view of the HP correspondence principle to find a phase of weakly-coupled long strings matching onto a near extremal black-hole geometry.

This crisis is solved with the help of a new phase of the string theory [21]. A phase in which the electric field is diminished by dynamically ionizing away fundamental strings off the bound state of D3-branes and F1-strings defining the NCOS. This is natural in view of the properties of the metric (3.10), as pointed out in [19], i.e. it is asymptotic to that of smeared F1-strings at large $u$.

The ionization of F1-strings replaces the *long* open-string dominance of the Hagedorn regime and it matches smoothly the supergravity regime at the appropriate temperature. According to (2.31) and (2.32), if the entropy of the 'stringy NCOS' phase is dominated by ionized F1-strings at $\lambda_o \ll 1$, we have

$$S_{\text{ionized}} \sim nLT \sim \frac{N^2 V_\perp L}{\theta_e \lambda_o} T\,. \tag{3.20}$$

This entropy law matches (3.19) precisely along the required HP transition line:

$$\lambda_o \sim \frac{1}{\theta_e u^2} \sim \left(\frac{T_H}{T}\right)^2. \tag{3.21}$$

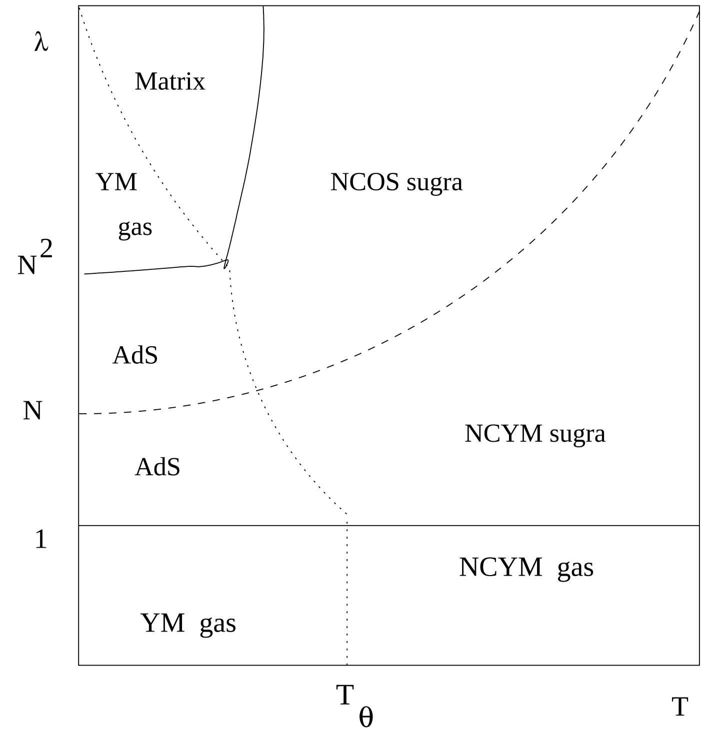


**Figure 1:** The four-dimensional phase diagram in NCYM variables. Full lines denote crossovers based on the correspondence principle. The dashed line is the S-duality transition. The dotted lines denote the onset of noncommutative effects and $T_\theta \equiv 1/\sqrt{\theta}$.

There is no sign of negative specific heat metrics in the supergravity phases, and indeed the natural matching to a system of free F1-strings indicates that the whole phase diagram can be studied within the canonical ensemble. These results are summarized in figures 1 and 2, where the phase of F1-string ionization is termed 'matrix', since it corresponds to the thermodynamics of matrix strings [32].

In the next subsection we pause to resurrect the long open strings by considering a very small but non-vanishing value of $\alpha'$. By this we show that, although such dregrees of freedom seem to be forbidden at $T_H = 1/\sqrt{\theta_e}$, they are not *a priori* discriminated against by such a type of analysis.

### 3.2 Regularizing the NCOS theory

According to our discussion in the preceding section, the absence of appropriate supergravity matching of Hagedorn phases is related to the absence of metrics with negative specific heat. These appear naturally in the case of D$p$-branes without F1-string charge. It is then interesting to consider a 'near NCOS' theory, defined by introducing a *finite*, albeit large hierachy between the Regge slope parameters $\alpha'$ and $\alpha'_e$. By considering the 'regularized NCOS' theory at finite $\alpha'$, we hope to see negative specific heat phases arising at sufficiently high energy. The thermodynamics of these phases can be studied in the microcanonical ensemble, as a function of the total

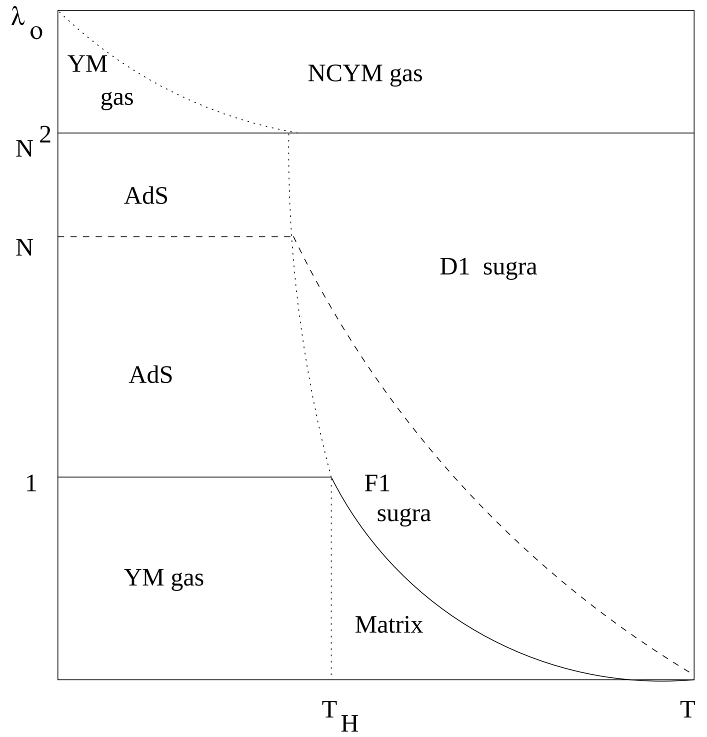


**Figure 2:** The four-dimensional phase diagram in NCOS variables. The conventions for the transition lines are the same as in figure 1. The notations 'F1 sugra' and 'D1 sugra' refer to the fact that the corresponding metrics are well approximated by those of smeared one-branes.

energy of the system, with the temperature as a derived quantity. Alternatively, we can use the horizon radius $r_0$ or the energy variable $u_0 = r_0/R^2$ as a control parameter of the microcanonical description. In the region of positive specific heat, $u_0 \sim T$, whereas we have $T \sim 1/r_0$ in the region of negative specific heat (the Schwarzschild regime).

As emphasized in section 1, phases with negative specific heat can be invisible in the canonical ensemble, and yet reappear in the microcanonical analysis. By studying the 'almost NCOS' theory with small but finite ratio $\alpha'/\alpha'_e$, we intend to see how the NCOS theory fits in the more general dynamics of the full (D3, F1) bound state, including all the phases that show up in the microcanonical analysis of the supergravity backgrounds. We find a consistent phase diagram with no place for a long-string phase with effective Hagedorn temperature $T_H \sim 1/\sqrt{\theta_e}$.

From the point of view of the supergravity solutions, keeping a finite $\alpha'$ implies considering the full (D3, D1) bound-state metric asymptotic to flat ten-dimensional space:

$$ds^2 = \frac{1}{\sqrt{H}}\left(-h dt^2 + dy^2 + \frac{f}{t_\theta^2} d\mathbf{x}^2\right) + \sqrt{H}\left(\frac{dr^2}{h} + r^2 d\Omega_5^2\right), \tag{3.22}$$

with dilaton and NS-background:

$$e^{2\tilde{\phi}} = \tilde{g}_s^2 f\,, \qquad 2\pi\tilde{\alpha}' B = \frac{f}{H}\,, \tag{3.23}$$

and profile functions

$$H = 1 + \frac{r_0^4 \sinh^2\alpha}{r^4}\,, \qquad h = 1 - \frac{r_0^4}{r^4}\,, \qquad f^{-1} = c_\theta^2 + \frac{s_\theta^2}{H}\,, \tag{3.24}$$

and

$$t_\theta \equiv \frac{\theta}{2\pi\tilde{\alpha}'}\,, \qquad c_\theta \equiv \frac{1}{\sqrt{1+t_\theta^2}}\,, \qquad s_\theta \equiv \frac{t_\theta}{\sqrt{1+t_\theta^2}}\,.$$

The parameter $\sinh\alpha$ is fixed in terms of the charge radius of the extremal solution:

$$\frac{r_0^4}{2}\sinh 2\alpha = \frac{\tilde{g}_s N}{c_\theta}\frac{(2\pi\sqrt{\tilde{\alpha}'})^4}{\mathrm{Vol}(S^5)} = R^4 = 2\lambda(\tilde{\alpha}')^2\,. \tag{3.25}$$

With these conventions, the NCOS limit is obtained by $t_\theta \to \infty$ and $\tilde{\alpha}' \to 0$ with $\tilde{G}_s = \tilde{g}_s/c_\theta$ fixed, without any further rescalings of the metric. Also, we set the usual $r = R^2 u$ to write the metric in terms of a radial coordinate with dimensions of energy. In this limit, the combination $f/t_\theta^2 \to \hat{f}$, and the noncommutativity length scale arises as $t_\theta^4 R^4 \to a_\theta^4 = R_o^4$.

We want to calculate the boundary line where the specific heat becomes infinite. This corresponds to the critical point beyond which the thermodynamics becomes Schwarzschild-like with negative specific heat. The formula for the inverse temperature $\beta = 1/T$ as a function of the Schwarzschild radius is

$$\beta = \frac{1}{\pi} r_0 \cosh\alpha\,. \tag{3.26}$$

We want to localize the turning point $d\beta/dr_0 = 0$. The dependence of $\sinh\alpha$ on $r_0$ may be determined by taking the derivative of (3.25):

$$\frac{d\sinh\alpha}{dr_0} = -\frac{2}{r_0}\frac{\sinh 2\alpha}{\cosh\alpha + \sinh^2\alpha}\,.$$

Inserting this back into the equation for $d\beta/dr_0 = 0$, we find a critical value $(\sinh\alpha)_{\rm critical} = \mathcal{O}(1)$, which leads to $(r_0)_{\rm critical} \sim R$. As expected, the crossover to the branch with negative specific heat occurs when the Schwarzschild radius is comparable to the charge radius, or

$$u_0 \sim T \sim \frac{1}{R}\,. \tag{3.27}$$

Since $R^4 \sim (\tilde{\alpha}')^2\lambda$, this line is

$$\lambda \sim \left(\frac{\tilde{T}_H}{T}\right)^4\,, \tag{3.28}$$

where we have defined the Hagedorn temperature of the 'magnetic' string theory:

$$\tilde{T}_H \equiv \frac{1}{\sqrt{8\pi^2\tilde{\alpha}'}}\,. \tag{3.29}$$

In NCOS variables it reads

$$\lambda_o \sim \left(\frac{T_H T_{\alpha'}}{T^2}\right)^2, \qquad \text{where } T_{\alpha'} \equiv \frac{1}{\sqrt{8\pi^2\alpha'}}\,. \tag{3.30}$$

In any set of variables, the important conclusion is that this line hits the NCOS correspondence line (3.21) at a temperature $T_{\alpha'}$. Therefore, once we keep $\alpha'$ finite and do not take the strict NCOS limit, we find that the matching of the NCOS phase to supergravity necessarily involves some matching to negative specific heat metrics. This suggests that the NCOS phase is actually bounded at very weak NCOS coupling and high temperature by a more or less standard Hagedorn phase with a Hagedorn temperature given by $T_{\alpha'}$. The 't Hooft coupling of the NCOS at this point is the maximum value compatible with a Hagedorn phase. It is given by

$$(\lambda_o)_{\max} \sim \left(\frac{T_H}{T_{\alpha'}}\right)^2 = \frac{\alpha'}{\alpha'_e} = \epsilon$$

and goes to zero in the NCOS limit.

The significance of the temperature $T_{\alpha'}$ from the point of view of the microscopic NCOS theory and the ionization mechanism of [21] is clear from equation (1.14). Namely, the effective Hagedorn temperature of the bound state rises as the system loses F1-charge. This process continues until all the electric field is depleated $\mathcal{E} = 0$, which implies $\alpha' = \alpha'_e$, so that the effective Hagedorn temperature at the end of the 'ionization' process is $T_{\alpha'}$. Beyond this point the system cannot escape the formation of long open strings and $T_{\alpha'}$ is a standard maximal temperature of a Hagedorn phase with negative specific heat.

A further check of the scenario comes from considering the curvature threshold. In gauge-theory variables, the Ricci curvature in string units at the horizon in the deep Schwarzschild regime is of order $\tilde{\alpha}'/r_0^2$. Thus, the Ricci curvature of the NCOS metric in the same regime is

$$\alpha'_e\,(\text{Ricci})_{\text{NCOS}} \sim \frac{\alpha'_e}{\tilde{\alpha}'}\left(\frac{\tilde{\alpha}'}{\alpha'_e}e^{\tilde{\phi}}\right)\cdot\tilde{\alpha}'\,(\text{Ricci})_{\text{NCYM}} \sim \tilde{g}_s\frac{\tilde{\alpha}'}{r_0^2} \sim \frac{\alpha'}{r_0^2}\,, \tag{3.31}$$

where we have used (3.11) and $e^{\tilde{\phi}} \sim \tilde{g}_s$ in the asymptotically flat region. On the other hand, the Hawking temperature of such a Schwarzschild brane scales like $T \sim 1/r_0$. Demanding the curvature to be of $\mathcal{O}(1)$ in string units gives a matching temperature for the correspondence line:

$$T_{\text{match}} \sim \frac{1}{r_0} \sim \frac{1}{\sqrt{\alpha'}} \sim T_{\alpha'}\,. \tag{3.32}$$

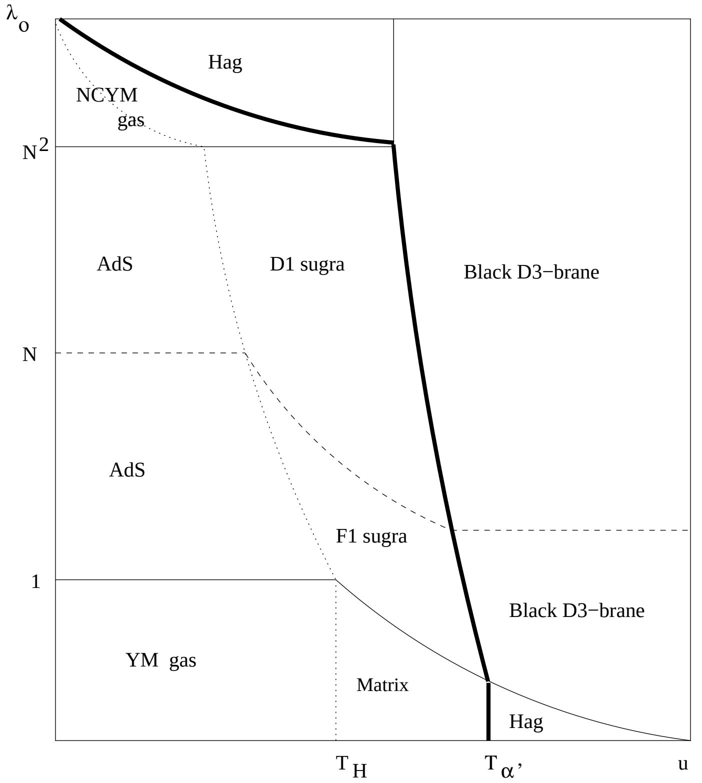


**Figure 3:** The phase diagram of the full (D3,F1) bound state in $\lambda_o$ versus the radial energy variable $u$, including the maximum temperature line that appears when the NCOS decoupling is not complete. To the right of the thick line one finds true Hagedorn phases or black-brane phases with negative specific heat. In the exact NCOS limit $T_{\alpha'} \to \infty$ and this thick line is pushed to infinite energies. The region of large energies $u > T_H$ and low NCOS coupling (below the dashed $S$-duality line) has universal features, independent of the D$p$-brane dimension.

Thus, the natural temperature of whatever phase matches the negative specific heat patch is $T_{\alpha'}$. This lends further support to the idea that $T_{\alpha'}$ is the Hagedorn temperature of the true phase of long open strings surviving in the high-energy corner. We collect the detailed structure of the 'almost NCOS' theory in figure 3.

Incidentally, it is interesting to note that (3.31) is exactly the same curve as (3.14), when written in terms of the running energy variable $u = r/R^2$. Namely (3.31) is the continuation of (3.14) past $u \sim T_{\alpha'}$. This makes the absence of a Hagedorn phase at $T < T_{\alpha'}$ rather dramatic, since we are supposed to match the *same* curve on both sides. This is a very specific property of smeared F1-string metrics, i.e. they have a stringy curvature threshold just like that of a Schwarzschild brane, and yet their thermodynamics is like that of a near-extremal brane.

### 3.3 Generalization to D$p$-branes with $p < 5$

Our discussion of the various supergravity phase diagrams can be readily generalized to $p \leq 5$ using the results of [19]. We avoid discussing the case $p = 5$ which has special features. The same qualitative behaviour as in the four-dimensional case is observed, provided we are sufficiently near the NCOS regime.

At low energies the D$p$ component of the bound state dominates the physics and the supergravity backgrounds are well approximated by those of near-extremal D$p$-brane metrics, dual to ordinary SYM in $p+1$ dimensions (cf. [45]), with HP transition line

$$\lambda_o \sim \left(\frac{T}{T_H}\right)^{3-p}, \qquad \text{for } T \ll T_H \,. \tag{3.33}$$

The full $(\mathrm{D}p_N, \mathrm{F1}_n)$ bound-state metrics are characterized by a length scale $R_o$, the charge radius, given by

$$R_o^{7-p} = \frac{(2\pi)^{6-p}}{(7-p)\,\mathrm{Vol}(S^{8-p})} \lambda_o (\alpha'_e)^{\frac{7-p}{2}} \,. \tag{3.34}$$

This scale also marks the onset of noncommutative dynamics or, in other words, the influence of the F1-string component of the bound state. For temperatures $TR_o \gg 1$ the supergravity solution is approximated by that of F1-strings, smeared over the $(p-1)$-dimensional 'transverse' volume $V_\perp$. Therefore, the HP transition line for $T \gg T_H$ is 'universal' in the sense that it does not depend on the D$p$-brane dimensionality:

$$\lambda_o \sim \left(\frac{T_H}{T}\right)^2, \qquad \text{for } T \gg T_H \,. \tag{3.35}$$

In addition, the classical thermodynamics of black-hole metrics is insensitive to the crossing of the charge radius by the horizon, i.e. the smeared F1-solutions exactly give the thermodynamic functions of pure D$p$-branes.

Therefore, the peculiar situation exposed in the example of the D3-brane generalizes to other dimensionalities. The large-$N$ thermodynamics of the supergravity phase is equivalent to that of ordinary SYM theory, i.e. the noncommutativity scale $\alpha'_e$ does not enter, except for setting the scale of the gauge coupling. What is very surprising is that this includes the region with temperatures larger than the Hagedorn temperature. On the other hand, the HP correspondence line for large curvature corrections is controlled by the approximate smeared F1-string metrics for $T \gg T_H$. And this threshold is characteristic of two-dimensional physics, universal with respect to the D$p$-brane dimensionality.

Thus, the same puzzle of the matching of Hagedorn long-strings remains for general values of $p$. The F1-ionization picture is naturally borne out as the right solution, provided the weak-coupling NCOS entropy is dominated by the F1-component at the supergravity matching line, a property that was argued in section 1.

At stronger coupling, the above supergravity description of the NCOS systems undergoes an $S$-duality transition. At low temperatures, the curve is

$$\lambda_o \sim N^{\frac{2(5-p)}{7-p}} \left(\frac{T}{T_H}\right)^{3-p}, \qquad \text{for } T \ll T_H \,, \tag{3.36}$$

whereas at high temperatures, the transition occurs around the line

$$\lambda_o \sim N^{\frac{5-p}{6-p}} \left(\frac{T_H}{T}\right)^{\frac{7-p}{6-p}}, \qquad \text{for } T \gg T_H \,. \tag{3.37}$$

Beyond these boundaries, the phase diagram of the NCOS systems is strongly dependent on the dimensionality.

The 'almost NCOS' versions, with finite ratio $\alpha'/\alpha'_e$, show similar features to the four-dimensional case. The NCOS phase consistent with the mechanism of F1-string ionization ends at temperature $T_{\alpha'}$, giving rise to a true Hagedorn phase of long open strings on the D$p$-brane. This matches the metric of Schwarzschild black-branes along the *same* curve (3.35), when written in microcanonical variables. The critical line where the canonical ensemble breaks down in the supergravity picture is

$$\lambda_o \sim \frac{T_{\alpha'}^{5-p} T_H^2}{T^{7-p}} \,. \tag{3.38}$$

**No F1-string ionization in the supergravity regime.** It is interesting to determine the thermal barrier for F1-string ionization in the supergravity regime for general values of $p$. Such a computation would shed light on the correct picture for the ionized F1-strings in transverse dimensions, namely whether they should be considered as totally spread in the transverse dimensions to the D$p$-brane, or rather clumping in a 'halo' in the proximity of the D$p$-brane bound state. Since we claim that the ionized bound state *together* with the F1-strings matches to the supergravity solution of smeared F1-strings, it would be odd to find that the supergravity solution is unstable against F1-string discharge. We find that indeed the supergravity solutions at finite temperature are stable and the ionized F1-strings at weak coupling should be viewed as a 'cloud' that falls behind the black-brane horizon, matching to the smeared F1-strings dissolved in the supergravity solution. It is also interesting, in view of our discussion of the possible recombination effect for $p = 4$, to determine possible qualitative differences as a function of the NCOS dimensionality.

We would like to compute the free energy gap by emission of a single F1-string. It can be calculated by lowering a probe F1-brane to the horizon and computing its world-volume action. This was done in [22] for the $S$-dual four-dimensional case. Alternatively, we can use the results of [19] for the chemical potential of the supergravity solution:

$$\mu_{\rm bs} = \frac{L}{2\pi\alpha'} \sin\hat\theta \tanh\hat\alpha \,. \tag{3.39}$$

where $\hat{\theta}$ is the control parameter of the NCOS limit, so that $\cos^2\hat{\theta} = \alpha'/\alpha'_e$, and $\hat{\alpha}$ is the rapidity angle controlling the departure from extremality, i.e.

$$\frac{1}{2}\sinh 2\hat{\alpha}\cos^2\hat{\theta} = \left(\frac{7-p}{4\pi}TR_o\right)^{\frac{2(p-7)}{5-p}} . \tag{3.40}$$

We can use this equation to solve for $\hat{\alpha}$ in the NCOS limit and obtain

$$\mu_{\rm bs} = \frac{L}{2\pi\alpha'} - 2\pi LT_H^2\left(1 + K'(\lambda_o t^{7-p})^{\frac{2}{5-p}}\right), \tag{3.41}$$

with $K'$ some $\mathcal{O}(1)$ constant. The liberated F1-strings at infinity have chemical potential

$$\mu_{\rm F1} = \frac{L}{2\pi\alpha'} - 2\pi LT^2 . \tag{3.42}$$

Thus, the supergravity analog of (2.27) is

$$\Delta F_{\rm ion} = \mu_{\rm F1} - \mu_{\rm bs} = 2\pi LT_H^2\left(1 + K'\left(\lambda_o t^{7-p}\right)^{\frac{2}{5-p}} - t^2\right). \tag{3.43}$$

We see that interaction effects tend to suppress the ionization process also in the supergravity regime. In fact, for $t \gg 1$, we have $\mu_{\rm bs} - \mu_{\rm F1} < 0$ for all $\lambda_o \gg 1/t^2$, i.e. there is no ionization throughout all the supergravity regime. On the other hand, the instability for F1-string emission appears precisely for $\lambda_o$ in the order of magnitude of the HP line $\lambda_o \lesssim 1/t^2$.

Therefore, the F1-strings ionized in the weak-coupling regime should be thought as a 'halo' of the bound-state that falls behind the horizon of the black-brane at the supergravity matching. This computation also shows that no special phenomenon occurs in the supergravity regime for $p = 4$. Thus, the matching to supergravity disfavours the possibility of having a recombination of the (D4, F1) bound state.

## 4. Conclusions

We have shown from several points of view that it is most likely that NCOS systems with $d < 5$ resort to their microscopic constituent picture as the temperature is raised towards a triple temperature: a temperature where noncommutative effects become important, where a Hagedorn transition may take place, and where the ionization process starts becoming operative.

Even if long open strings on the NCOS bound state give the highest density of states in the high-energy regime, consistency with the correspondence principle of [15] forces upon us the ionization picture drawn from the canonical ensemble analysis, where the nominal Hagedorn temperature is surpassed without ever exciting a significant number of long open strings. In this picture, the entropy is carried by massless excitations and soon becomes dominated by two-dimensional fields, which in turn satisfy the appropriate matching to the supergravity description. The case $d = 5$ is special because a first-order phase transition stops the ionization of F1-strings within the weak-coupling regime, so that the canonical ensemble has no graceful exit into the supergravity regime. If for some unknown reason the long open strings were activated precisely at $d = 5$, they would erase the first-order phase transition but stop ionization anyway, so that the system still fails the correct matching to supergravity. Hence, the tension between the correspondence principle and our pictures of the weak-coupling dynamics leaves us with a genuine puzzle for the case of five-dimensional NCOS theories.

The possibility that long open strings dominate the thermodynamics in a transient regime that is invisible in the canonical ensemble (in analogy with the case of AdS thermodynamics) is unlikely since the required strong-coupling phases in the gravity picture are not found.

In systems containing extended objects and only nonlocal observables, it may well be that there are cases when a transition between a microcanonical ensemble and a canonical one is very complex. This does not seem to occur in the AdS/CFT case, but perhaps it occurs for NCOS. In such a case one could imagine that the microcanonical and canonical ensembles somehow sample totally disconnected regions of configuration space at very high energy. Namely, forcing the temperature to be above Hagedorn $T > T_H$ constrains the system to proceed through the ionization mechanism, because configurations with long strings that maximize the entropy necessarily have $T \approx T_H$. In principle, it is possible that the system has two different high-energy limits, with totally different behaviour depending on whether we impose canonical or microcanonical boundary conditions in the thermal ensemble. According to this *ad hoc* picture, the phase of long open strings with entropy $S \approx E/T_H$ would extend to arbitrarily high energies, i.e. it would resemble the transient picture considered above, but with the return to the positive specific heat behaviour only occuring at infinite energy.

We find also this escape hatch unlikely on the basis of particular examples where the NCOS system is $S$-dual to ordinary field theories. One such example is the two-dimensional case, where the NCOS theory based on the $(N, n)$ bound state is $S$-dual to ordinary $\mathrm{U}(n)$ SYM with $N$ units of electric flux [46, 23]. Another example in four dimensions is that of *rational* NCOS theories. Namely, working in finite commutative volume $V_\perp$ we have a finite number, $n$, of F1-strings melted in the bound state. Then this theory is $S$-dual to NCYM with rational dimensionless theta parameter

$$\Theta \equiv \frac{2\pi\theta}{V_\perp} = \frac{N}{n}\,. \tag{4.1}$$

With relatively prime $N$ and $n$, this theory is in turn the Morita-dual of *ordinary* $\mathrm{U}(n)$ SYM with some units of 't Hooft magnetic flux [31, 47, 24], living on a smaller volume $LV_\perp/n^2$. In this representation, it is clear that the extreme high-energy asymptotics of this theory cannot be of Hagedorn type, independently of whether we use the canonical or the microcanonical thermal ensembles. Although this argument does not exclude the possibility of a transient regime of Hagedorn density of states, it does exclude the exotic possibility noted above where the long-string phase would extend to infinite energies.

Based on these considerations, we conclude that weakly coupled, gravity free, long-string picture is not microscopic but only effective. Its validity seems to melt away near the potential Hagedorn transition. At this point we are remainded of the fact that open NCOS strings are not BPS objects. Since the NCOS limit involves $g_s \to \infty$, the general validity of our parametrization of the NCOS dynamics can be called into question in extreme situations. The thermodynamics at Hagedorn temperatures seems to be one of these situations.

One may be tempted to turn the argument around and say that this picture is what would occur in the case of small but finite coupling in all string theories. In any case, the model we have analysed is an explicit example of constituent 'deconfinement'.

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